

$$[2] \lim_{x \rightarrow 1^+} \cos(f(x)-3) = \cos(\lim_{x \rightarrow 1^+} (f(x)-3))$$

WATCH OUT FOR

LIMITS FROM ONE SIDE ONLY

SINCE $\cos x$ IS CONTINUOUS EVERYWHERE

$$\lim_{x \rightarrow 1^+} (f(x)-3) = \lim_{x \rightarrow 1^+} f(x) - \lim_{x \rightarrow 1^+} 3 = 4 - 3 = 1$$

$$\text{so } \lim_{x \rightarrow 1^+} \cos(f(x)-3) = \cos 1$$

$$\lim_{x \rightarrow 1^-} \cos(f(x)-3) = \cos(\lim_{x \rightarrow 1^-} (f(x)-3))$$

SINCE $\cos x$ IS CONTINUOUS EVERYWHERE

$$\lim_{x \rightarrow 1^-} (f(x)-3) = \lim_{x \rightarrow 1^-} f(x) - \lim_{x \rightarrow 1^-} 3 = 2 - 3 = -1$$

$$\text{so } \lim_{x \rightarrow 1^-} \cos(f(x)-3) = \cos(-1) = \cos 1$$

SINCE $\cos(-x) = \cos x$ FOR ALL x

$$\text{so } \lim_{x \rightarrow 1} \cos(f(x)-3) = \cos 1$$

[3] $\arctan x$ IS CONTINUOUS EVERYWHERE | (1/2)

$1 + \ln x$ IS CONTINUOUS AT ALL $x > 0$ | (1)

SO $\frac{\arctan x}{1 + \ln x}$ IS CONTINUOUS AT ALL $x > 0$ | (1)

EXCEPT WHERE $1 + \ln x = 0$ | (1)

$$\ln x = -1$$

$$x = e^{-1} = \frac{1}{e} | (1)$$

$$[4] \lim_{x \rightarrow -3} \frac{\frac{27}{x^2} - \frac{18}{x^2+x}}{x+3} = \boxed{\textcircled{1} \lim_{x \rightarrow -3} \frac{\frac{27}{x^2} - \frac{18}{x(x+1)}}{x+3} \cdot \frac{x^2(x+1)}{x^2(x+1)}}$$

$$\frac{\frac{27}{9} - \frac{18}{9-3}}{-3+3} \rightarrow \frac{3-3}{0} \rightarrow \frac{0}{0}$$

$$= \lim_{x \rightarrow -3} \frac{27(x+1) - 18x}{x^2(x+1)(x+3)}$$

$$\textcircled{1} \lim_{x \rightarrow -3} \frac{9x+27}{x^2(x+1)(x+3)} = \lim_{x \rightarrow -3} \frac{9(x+3)}{x^2(x+1)(x+3)}$$

$$\textcircled{\frac{1}{2}} \lim_{x \rightarrow -3} \frac{9}{x^2(x+1)} = \frac{9}{9(-2)} = \boxed{-\frac{1}{2}} \textcircled{\frac{1}{2}}$$

$$\lim_{x \rightarrow -3} \sin^{-1} \frac{\frac{27}{x^2} - \frac{18}{x^2+x}}{x+3} = \boxed{\textcircled{\frac{1}{2}} \sin^{-1} \left(-\frac{1}{2}\right)} = \boxed{-\frac{\pi}{6}} \textcircled{\frac{1}{2}}$$

SINCE $\sin^{-1} x$ IS CONTINUOUS

$\textcircled{\frac{1}{2}}$ AT ALL x SUCH THAT $-1 < x < 1$

[5] BQ IS CORRECT $f(x) = \frac{4x}{3+2x-x^2}$ HAS INFINITE

DISCONTINUITIES WHERE $3+2x-x^2=0$

$$\text{IE. } (3-x)(1+x)=0$$

$x=3$ OR $x=-1$ IN $(-2, 4)$

SO f IS NOT CONTINUOUS ON $(-2, 4)$ AND IVT DOES NOT APPLY

$$[6][a] \lim_{x \rightarrow 1} \frac{2x^2 - 3x - 2}{2 + x - x^2} = \frac{\lim_{x \rightarrow 1} (2x^2 - 3x - 2)}{\lim_{x \rightarrow 1} (2 + x - x^2)} \quad \left(\frac{1}{2}\right)$$

$$= \frac{\lim_{x \rightarrow 1} 2x^2 - \lim_{x \rightarrow 1} 3x - \lim_{x \rightarrow 1} 2}{\lim_{x \rightarrow 1} 2 + \lim_{x \rightarrow 1} x - \lim_{x \rightarrow 1} x^2}$$

$$= \frac{(\lim_{x \rightarrow 1} 2)(\lim_{x \rightarrow 1} x)(\lim_{x \rightarrow 1} x) - (\lim_{x \rightarrow 1} 3)(\lim_{x \rightarrow 1} x) - 2}{2 + 1 - (\lim_{x \rightarrow 1} x)(\lim_{x \rightarrow 1} x)}$$

$$= \frac{2 \cdot 1 \cdot 1 - 3 \cdot 1 - 2}{2 + 1 - 1 \cdot 1} = \frac{2 - 3 - 2}{2 + 1 - 1} = \frac{-3}{2} \quad \left(\frac{1}{2}\right)$$

MUST SHOW
ALL FACTORS

[b] $f(2) = -\frac{5}{3}$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{\sqrt{4x+1} - \sqrt{x+7}}{4-2x} \cdot \frac{\sqrt{4x+1} + \sqrt{x+7}}{\sqrt{4x+1} + \sqrt{x+7}} \quad (1)$$

MUST HAVE
"lim+" ON

EACH LINE
AS SHOWN

$$\frac{\sqrt{9} - \sqrt{9}}{4-4} \rightarrow \frac{0}{0}$$

$$= \lim_{x \rightarrow 2^+} \frac{4x+1 - (x+7)}{-2(x-2)(\sqrt{4x+1} + \sqrt{x+7})}$$

$$= \lim_{x \rightarrow 2^+} \frac{3x-6}{-2(x-2)(\sqrt{4x+1} + \sqrt{x+7})} \quad (1/2)$$

$$= \lim_{x \rightarrow 2^+} \frac{3(x-2)}{-2(x-2)(\sqrt{4x+1} + \sqrt{x+7})}$$

$$= \lim_{x \rightarrow 2^+} \frac{3}{-2(\sqrt{4x+1} + \sqrt{x+7})} = \frac{3}{-2(\sqrt{9} + \sqrt{9})} = \frac{3}{-2(3+3)}$$

$$\frac{1}{2} = \frac{3}{-2(6)} = -\frac{1}{4} \quad (1/2)$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{2x^2 - 3x - 2}{2 + x - x^2} = \lim_{x \rightarrow 2^-} \frac{(x-2)(2x+1)}{(x-2)(-x-1)} \quad (1)$$

MUST HAVE
"lim-" ON
EACH LINE AS SHOWN

$$\frac{8-6-2}{2+2-4} \rightarrow \frac{0}{0}$$

$$= \lim_{x \rightarrow 2^-} \frac{2x+1}{-x-1} = \frac{4+1}{-2-1} = -\frac{5}{3} \quad (1/2)$$

$\frac{1}{2}$ $\lim_{x \rightarrow 2^+} f(x)$ AND $\lim_{x \rightarrow 2^-} f(x)$ EXIST, BUT DO NOT EQUAL $\frac{1}{2}$

SO f HAS A JUMP DISCONTINUITY AT $x=2$

AND f IS NOT CONTINUOUS AT $x=2$ SINCE $\lim_{x \rightarrow 2} f(x)$

$\frac{1}{2}$

DOES NOT EXIST

$\frac{1}{2}$